

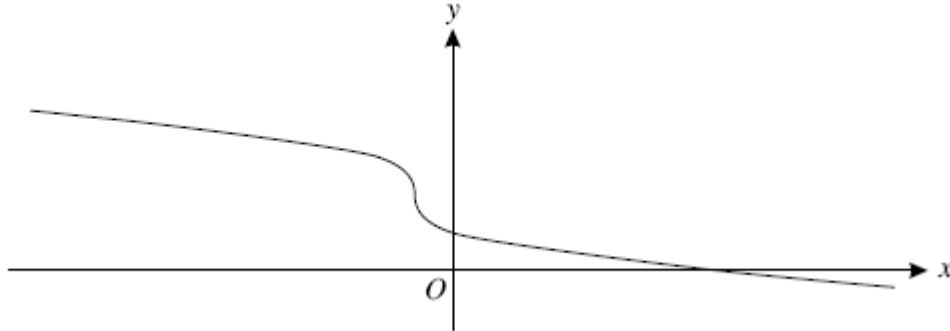
C3 Functions

1. [June 2010 qu.9](#)

The functions f and g are defined for all real values of x by $f(x) = 4x^2 - 12x$ and $g(x) = ax + b$, where a and b are non-zero constants.

- (i) Find the range of f . [3]
- (ii) Explain why the function f has no inverse. [2]
- (iii) Given that $g^{-1}(x) = g(x)$ for all values of x , show that $a = -1$. [4]
- (iv) Given further that $gf(x) < 5$ for all values of x , find the set of possible values of b . [4]

2. [Jan 2010 qu.4](#)



The function f is defined for all real values of x by $f(x) = 2 - \sqrt[3]{x+1}$.

The diagram shows the graph of $y = f(x)$.

- (i) Evaluate $ff(-126)$. [2]
- (ii) Find the set of values of x for which $f(x) = |f(x)|$. [2]
- (iii) Find an expression for $f^{-1}(x)$. [3]
- (iv) State how the graphs of $y = f(x)$ and $y = f^{-1}(x)$ are related geometrically. [1]

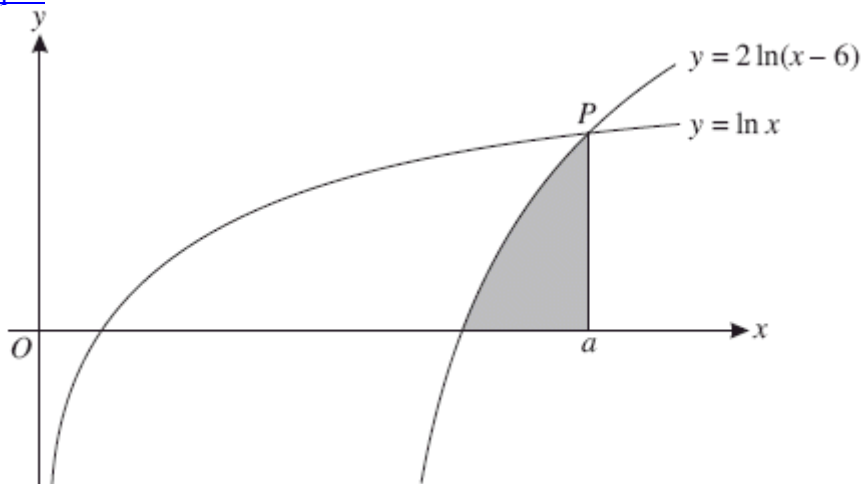
3. [June 2009 qu.5](#)

The functions f and g are defined for all real values of x by $f(x) = 3x - 2$ and $g(x) = 3x + 7$.

Find the exact coordinates of the point at which

- (i) the graph of $y = fg(x)$ meets the x -axis, [3]
- (ii) the graph of $y = g(x)$ meets the graph of $y = g^{-1}(x)$, [3]
- (iii) the graph of $y = |f(x)|$ meets the graph of $y = |g(x)|$. [4]

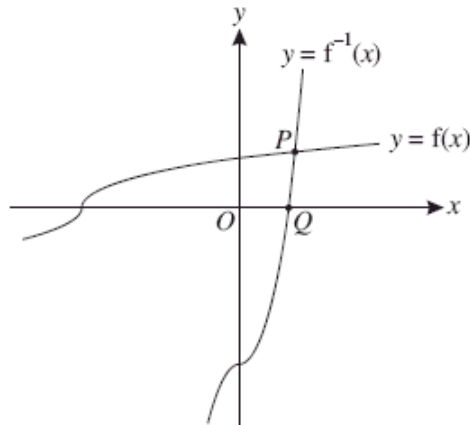
4. [June 2009 qu.8](#)



The diagram shows the curves $y = \ln x$ and $y = 2 \ln(x - 6)$. The curves meet at the point P which has x -coordinate a . The shaded region is bounded by the curve $y = 2 \ln(x - 6)$ and the lines $x = a$ and $y = 0$.

- (i) Give details of the pair of transformations which transforms the curve $y = \ln x$ to the curve $y = 2 \ln(x - 6)$. [3]
- (ii) Solve an equation to find the value of a . [4]

5. [Jan 2009 qu.6](#)



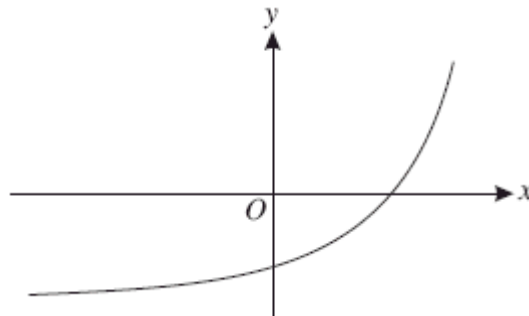
The function f is defined for all real values of x by $f(x) = \sqrt[3]{\frac{1}{2}x + 2}$.

The graphs of $y = f(x)$ and $y = f^{-1}(x)$ meet at the point P , and the graph of $y = f^{-1}(x)$ meets the x -axis at Q (see diagram).

(i) Find an expression for $f^{-1}(x)$ and determine the x -coordinate of the point Q . [3]

(ii) State how the graphs of $y = f(x)$ and $y = f^{-1}(x)$ are related geometrically, and hence show that the x -coordinate of the point P is the root of the equation $x = \sqrt[3]{\frac{1}{2}x + 2}$. [2]

6. [Jan 2009 qu.7](#)



The diagram shows the curve $y = e^{kx} - a$, where k and a are constants.

(i) Give details of the pair of transformations which transforms the curve $y = e^x$ to the curve $y = e^{kx} - a$. [3]

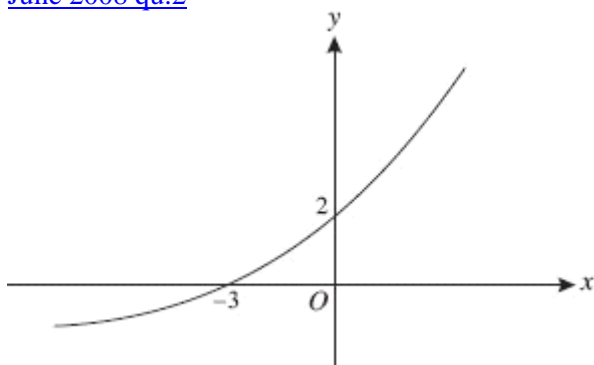
(ii) Sketch the curve $y = |e^{kx} - a|$. [2]

(iii) Given that the curve $y = |e^{kx} - a|$ passes through the points $(0, 13)$ and $(\ln 3, 13)$, find the values of k and a . [4]

7. [June 2008 qu.1](#)

Find the exact solutions of the equation $|4x - 5| = |3x - 5|$. [4]

8. [June 2008 qu.2](#)



The diagram shows the graph of $y = f(x)$. It is given that $f(-3) = 0$ and $f(0) = 2$. Sketch, on separate diagrams, the following graphs, indicating in each case the coordinates of the points where the graph crosses the axes:

- (i) $y = f^{-1}(x)$, [2]
 (ii) $y = -2f(x)$. [3]

9. [June 2008 qu.7](#)

It is claimed that the number of plants of a certain species in a particular locality is doubling every 9 years. The number of plants now is 42. The number of plants is treated as a continuous variable and is denoted by N . The number of years from now is denoted by t .

- (i) Two equivalent expressions giving N in terms of t are

$$N = A \times 2^{kt} \quad \text{and} \quad N = Ae^{mt}.$$
 Determine the value of each of the constants A , k and m . [4]
 (ii) Find the value of t for which $N = 100$, giving your answer correct to 3 significant figures. [2]
 (iii) Find the rate at which the number of plants will be increasing at a time 35 years from now. [3]

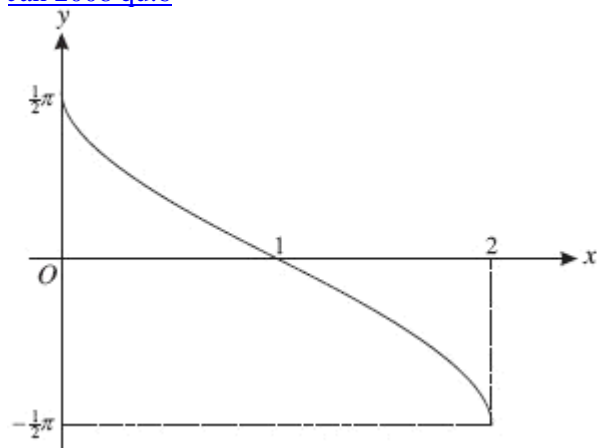
10. [Jan 2008 qu.1](#)

Functions f and g are defined for all real values of x by $f(x) = x^3 + 4$ and $g(x) = 2x - 5$.

Evaluate

- (i) $fg(1)$, [2]
 (ii) $f^{-1}(12)$. [3]

11. [Jan 2008 qu.6](#)



The diagram shows the graph of $y = -\sin^{-1}(x - 1)$.

- (i) Give details of the pair of geometrical transformations which transforms the graph of $y = -\sin^{-1}(x - 1)$ to the graph of $y = \sin^{-1} x$. [3]
 (ii) Sketch the graph of $y = \left| -\sin^{-1}(x - 1) \right|$. [2]
 (iii) Find the exact solutions of the equation $\left| -\sin^{-1}(x - 1) \right| = \frac{1}{3}\pi$. [3]

12. [June 2007 qu.2](#)
Solve the inequality $|4x - 3| < |2x + 1|$. [5]

13. [June 2007 qu.3](#)
The function f is defined for all non-negative values of x by $f(x) = 3 + \sqrt{x}$.
(i) Evaluate $ff(169)$. [2]
(ii) Find an expression for $f^{-1}(x)$ in terms of x . [2]
(iii) On a single diagram sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$, indicating how the two graphs are related. [3]

14. [June 2007 qu.5](#)
A substance is decaying in such a way that its mass, m kg, at a time t years from now is given by the formula $m = 240e^{-0.04t}$
(i) Find the time taken for the substance to halve its mass. [3]
(ii) Find the value of t for which the mass is decreasing at a rate of 2.1 kg per year. [4]

15. [Jan 2007 qu.9](#)
Functions f and g are defined by

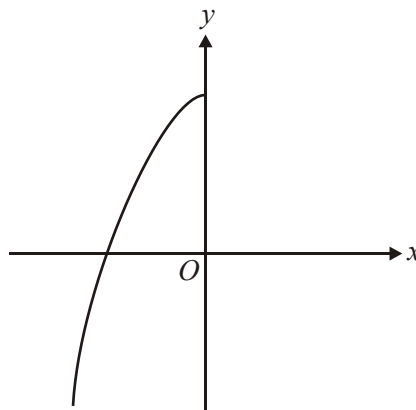
$$f(x) = 2 \sin x \quad \text{for } \frac{1}{2}\pi \leq x \leq \frac{3}{2}\pi,$$

$$g(x) = 4 - 2x^2 \quad \text{for } x \in \mathbb{R}.$$

 (i) State the range of f and the range of g . [2]
 (ii) Show that $gf(0.5) = 2.16$, correct to 3 significant figures, and explain why $fg(0.5)$ is not defined. [4]
 (iii) Find the set of values of x for which $f^{-1}g(x)$ is not defined. [6]

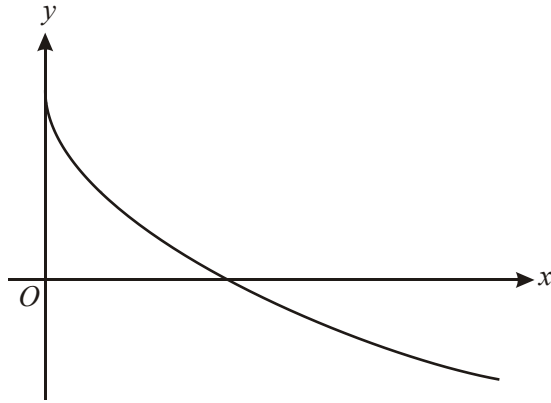
16. [June 2006 qu.2](#)
Solve the inequality $|2x - 3| < |x + 1|$. [5]

17. [June 2006 qu.6](#)



The diagram shows the graph of $y = f(x)$, where $f(x) = 2 - x^2$, $x \leq 0$.
 (i) Evaluate $ff(-3)$. [3]
 (ii) Find an expression for $f^{-1}(x)$. [3]
 (iii) Sketch the graph of $y = f^{-1}(x)$. Indicate the coordinates of the points where the graph meets the axes. [3]

18. [Jan 2006 qu.4](#)



The function f is defined by $f(x) = 2 - \sqrt{x}$ for $x \geq 0$. The graph of $y = f(x)$ is shown above.

- (i) State the range of f . [1]
- (ii) Find the value of $ff(4)$. [2]
- (iii) Given that the equation $|f(x)| = k$ has two distinct roots, determine the possible values of the constant k . [2]

19. [June 2005 qu.1](#)

The function f is defined for all real values of x by

$$f(x) = 10 - (x + 3)^2.$$

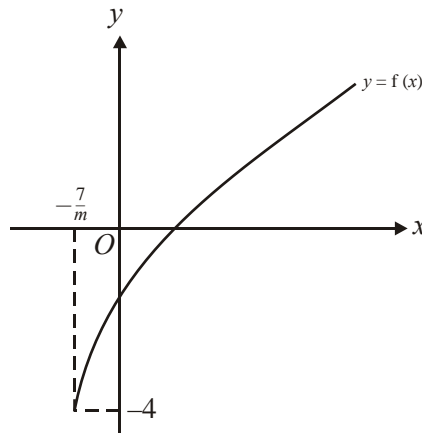
- (i) State the range of f . [1]
- (ii) Find the value of $ff(-1)$. [3]

20. [June 2005 qu.2](#)

Find the exact solutions of the equation $|6x - 1| = |x - 1|$.

[4]

21. [June 2005 qu.9](#)



The function f is defined by $f(x) = \sqrt{mx + 7} - 4$, where $x \geq -\frac{7}{m}$ and m is a positive constant.

The diagram shows the curve $y = f(x)$.

- (i) A sequence of transformations maps the curve $y = \sqrt{x}$ to the curve $y = f(x)$. Give details of these transformations. [4]
- (ii) Explain how you can tell that f is a one-one function and find an expression for $f^{-1}(x)$. [4]
- (iii) It is given that the curves $y = f(x)$ and $y = f^{-1}(x)$ do not meet. Explain how it can be deduced that neither curve meets the line $y = x$, and hence determine the set of possible values of m . [5]